QCD vacuum structure

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Several issues related to the structure of the QCD vacuum are reviewed. We concentrate mostly on results concerning instantons and center vortices.

1. Introduction

The structure of the QCD vacuum has been the subject of many lattice investigations over the years. Two phenomena have attracted most attention: chiral symmetry breaking and confinement. Instantons have been conjectured to play a key role in driving the former [1]. Here the lattice has entered the game by trying to provide non-perturbative information on the instanton ensemble. Activity in this field has already been reviewed in [2,3] and there is not much new to add this year. This is far from implying that we have reliably obtained all the information concerning QCD instanton dynamics. Fundamental issues like the size distribution or density of instantons are still not really settled, a point that will be briefly discussed in section 3.

Instantons have also come back on stage due to the beautiful instanton-monopole link arising at finite temperature [4,5]. This re-establishes some equality between these, a priori, two very different objects; instantons are made up of monopoles but monopoles can also be viewed as periodic arrays of instantons [6].

Monopoles bring us to the most popular scenario for confinement, the dual superconductor picture [7], which is beautifully at work in SUSY gauge theories [8]. 't Hooft's proposal of abelian projection [9] has been the subject of extensive lattice studies. The strongest version of this approach, based on abelian dominance, has been criticised in several respects [10–12]. Different abelian projections do not seem to be equivalent. Should one of them be preferred, it would imply that a particular U(1) is selected. The confining flux tube should then be abelian in nature and fields neutral with respect to it would be unconfined. A naive particular prediction would be no area law for adjoint Wilson loops. Although such loops are indeed screened at large distances, numerical investigations indicate the existence of a regime where they exhibit a confining behaviour with approximate Casimir scaling [13]. Moreover, [10] one also observes the more 'fundamental' center dominance, where instead of U(1) the relevant dynamical variables are assumed to be the ones in the center of the gauge group. Note that center dominance does not a priori solve the problem of Casimir scaling (adjoint fields are blind to the center). Although in [10] this was intended as a criticism, indications towards the possible relevance of center vortices to confinement arose from further work on the subject [14,15]. This has boosted the revival of a proposal of confinement that for very long remained asleep [16,17]. It is based on the fact that center vortices, and not U(1) monopoles, are the 'confining' configurations. Most of the activity during the past year has been devoted to this subject. I think it would be unfair to review QCD vacuum structure without acknowledging what has captured most of the attention. I will thus present my inexpert view on vortices in section 4.

Results concerning the dual superconductor approach to confinement will not be reviewed here (for recent reviews see [12,18]). Let me only mention that there is some agreement towards the fact that abelian dominance is indeed not really the issue [12,18]. Possible resolutions of the puzzles mentioned before have been proposed. An important result is that one can study condensation of monopoles by constructing a monopole creation operator [19]. The vev of such an operator is a disorder parameter for confinement, irrespective of the abelian projection used to define it. This is considered as a strong indication in favour of dual superconductivity. Related work concerning condensation of magnetic flux will be discussed in section 4.

The review is structured as follows. I will first concentrate on the less speculative phenomena, in particular the calculation of the topological susceptibility and the η' mass. This will be done in section 2. Section 3 presents a few other topics related to instantons, mostly concentrating on the instanton-monopole connection at finite temperature [4,5]. In section 4 results concerning center vortices will be presented.

2. χ and the η' mass

A lot of work has been devoted over the past years to computing the quenched topological susceptibility on the lattice. In the limit of large number of colours, it is related to the η' mass through the Witten-Veneziano formula [20],

$$\chi_q = \frac{\langle Q^2 \rangle}{V} = \frac{f_\pi^2}{2N_f}\,(m_\eta^2 + m_{\eta'}^2 - 2m_K^2) \quad . \tag{1}$$

Measuring the fluctuations of topological charge on the lattice has turned out to be a difficult enterprise (for a detailed description see [2]). I believe, by now, we can safely say that χ_q has been successfully determined both for SU(2) and SU(3). Continuum extrapolations of the available lattice results give (taking $\sqrt{\sigma} = 440\text{MeV}$) [2]

SU(2) $\quad \chi_q = (214 \pm 18\text{MeV})^4$

SU(3) $\quad \chi_q = (200 \pm 18\text{MeV})^4$

in pretty good agreement with the large N prediction $\chi_q \sim (180\text{MeV})^4$.

The situation is different for the unquenched susceptibility. In the chirally broken phase

$$\chi = \frac{f_\pi^2 m_\pi^2}{2N_f} + O(m_\pi^4) \propto m_q \quad , \tag{2}$$

in contrast to the behaviour in the symmetric phase where we expect $\chi \propto m_q^{N_f}$. The susceptibility is hence an observable clearly exhibiting the effect of dynamical fermions. The situation as of Latt'99 did not, however, look that promising. Available results from CP-PACS and the Pisa group [2] failed to see any chiral behaviour and exhibited an unquenched susceptibility independent of the quark mass. Results from UKQCD were more encouraging; the expected m_π dependence was indeed observed, although the value of f_π extracted from the slope of the susceptibility turned out about 20% below the physical value.

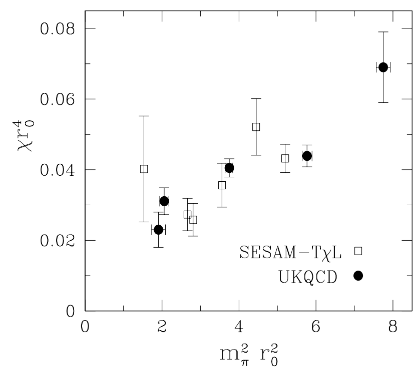

Figure 1. Topological susceptibility χr_0^4 vs $m_\pi^2 r_0^2$. Comparison between data from [21] and [23]

New results from UKQCD [21], the Pisa group [22] and the SESAM-TχL collaboration [23] are available. They use respectively $N_f = 2$ clover improved, staggered and Wilson fermions. In Fig. 1 a comparison between UKQCD and SESAM-TχL's data for χr_0^4 vs $m_\pi^2 r_0^2$ is presented (with the scale set by Sommer's $r_0 \simeq 0.49$ fm). Good agreement is observed. From a fit to eq. (2) keeping terms in m_π^4, UKQCD quotes a value $f_\pi = 105 \pm 5^{+18}_{-10}\text{MeV}$, in very good agreement with the expected physical value $f_\pi \simeq 93\text{MeV}$.

New results by CP-PACS also indicating the expected chiral behaviour have been presented in S. Aoki's plenary talk at this conference.

Still, the data from the Pisa group remain puzzling. We present them in Fig. 2. The errors are rather large, hence it is difficult to judge whether the result is really inconsistent with the expected chiral behaviour. A fit independent of m_q gives $\chi = (163 \pm 6\text{MeV})^4$ with a $chi^2/dof \sim 0.37$, while a fit with a linear homogeneous dependence in m_q

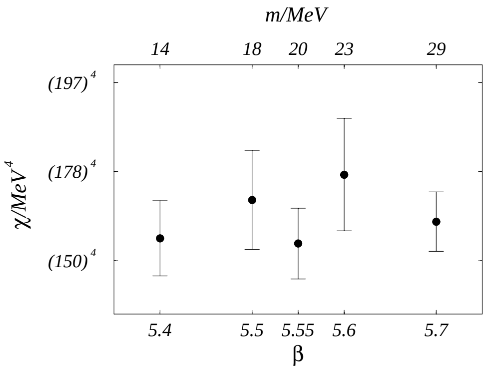

Figure 2. χ vs quark mass from [22].

gives $chi^2/dof \sim 0.94$. The largest β point, which seems the most problematic, might be affected by finite size effects which are rather severe for staggered fermions [24]. Taking it out improves the quality of the linear fit to $chi^2/dof \sim 0.3$. It is in any case clear that improved statistics and lighter quark masses are mandatory before deciding if these data represent a problem.

To finish this section let me mention some results by SESAM-TχL concerning the relation of the η' mass to topology [25]. Direct evaluation of the η' mass turns out to be very tough because it involves OZI suppressed ('disconnected') diagrams. Although improved estimators for these diagrams would be required, the connection with topology is already clearly exposed in [25]. The ratio between disconnected and connected diagrams is computed in sub-ensembles of Monte Carlo configurations characterised by their topological charge. A clear correlation between this ratio and the charge is observed. In particular, the disconnected piece vanishes in the sub-ensemble with charge $|Q| \leq 1.5$, indicating the degeneracy of the singlet and non-singlet states in the trivial topological charge sector.

Results from CP-PACS on the η' mass have also been presented in S. Aoki's plenary talk.

3. Instanton constituents

Looking at instantons as composite objects is not a new idea. It is indeed at work in the non-linear O(3) model in two dimensions - see for instance [26]- where each instanton seems to be composed of a pair of opposite Coulomb charges. It is the melting of instantons into a plasma of charges that is argued to induce the mass gap. In the deconfined phase charges remain bounded in dipoles and long range forces are screened. Similar ideas were put forward for QCD in [27] with fractional topological charge configurations, the merons, as fundamental objects. Merons have a singular action density and turn out analytically intractable. However, non-singular self-dual objects with fractional topological charge exist on a torus with twisted boundary conditions. They have indeed been advocated as relevant for confinement [28] and in some sense they can be seen as the fundamental constituents of some integer charge instantons.

Recently this constituent nature of instantons has been explicitly exposed [4,5]. The most general finite temperature instanton, the so-called caloron, has been constructed. It can be considered as a periodic array of instantons. For instanton size of the order of their separation, each instanton splits into N constituent BPS monopoles. The splitting takes place when the Polyakov loop at spatial infinity is non-trivial. Let me, for the sake of simplicity, concentrate on the SU(2) case. At spatial infinity the Polyakov loop is constant and parametrised by its trace: $2\cos(2\pi w)$. The masses of the constituent monopoles are determined by w and $\frac{1}{2}-w$. The standard Harrington-Shepard caloron with $w=0$ [29] is associated to a massive monopole at the center of the caloron plus a massless delocalised one at infinity. It is worth mentioning that the monopoles are located where the Polyakov loop has two degenerate eigenvalues [6]. This naturally makes contact with abelian projections. In a suitable gauge, out of the N monopoles, $\mathrm{N}-1$ are BPS static and only one is non-static. It is this one which gives topological charge to the caloron. Even more, the fermionic zero mode of the $Q=1$ caloron is precisely localised on the non-static monopole [30].

This constituent nature of calorons has already turned out to be crucial in solving a long standing controversy in SUSY Yang-Mills [31]. Calculations of the gluino condensate based on a direct semiclassical approximation did not agree with a 'weak' coupling expression with matter fields added (and then decoupled) to ensure calculability. Inclusion of the constituent monopole contributions in the semiclassical expansion has brought the two expressions to an agreement.

The way to make the constituent monopoles pop-up out of lattice calorons is by enforcing a non-trivial Polyakov loop at the spatial boundary. This is most naturally achieved by introducing twisted boundary conditions [32]. Indeed, it can be seen that the SU(2) BPS monopole with mass $4\pi^2/\beta$ precisely corresponds with the $Q = 1/2$ finite temperature fractional instanton (allowed to have non-zero magnetic charge due to the twisted b.c.). Non-triviality of the Polyakov loop can also be achieved by freezing the time links at the spatial boundary. This has been used in [33] to investigate the relevant configurations in finite temperature SU(2) (extracted by cooling). The trace of the boundary Polyakov loop is frozen to be zero below T_c, while it is fixed, above T_c, to the observed loop average. In the confined phase, calorons made up of two charge-1/2 monopoles dominate. Above T_c such monopoles are still present but coming in pairs of opposite topological charge. Dominance of $Q = 1/2$ objects for SU(2) at $T = 0$ has also been observed by imposing magnetic twisted boundary conditions [28]. Of course the relevant dynamical question, in particular below T_c, is whether such dominance survives the thermodynamic limit irrespective of the boundary conditions used.

The highly non-trivial behaviour of calorons arises due to the strong overlap in the periodic array of instantons . Overlap effects are also important at $T = 0$. As shown in [34] the action density of overlapping instantons differs considerably from the simple addition of single instanton profiles. Consequences for the extraction of the instanton size distribution from the lattice are important. In particular, when instantons are parallel oriented in colour space, large instantons are systematically missed by instanton finders. This is not an irrelevant issue since a large instanton component has been argued to give rise to confining behaviour for the Wilson loop [35]. It is also relevant for the instanton liquid model [1] since it indicates a possible failure of the 1-instanton approximation in ensembles with densities analogous to the ones obtained in lattice simulations.

A few other works have dealt with this picture of instanton constituents although from a different point of view. Ref. [36] presents a low-energy effective action for QCD that incorporates θ dependence. It can be described in terms of a Coulomb gas of fractionally charged objects, resembling the constituent monopoles described above. Also merons have come back on stage in [37] where regularised lattice merons and their fermionic zero modes have been obtained. Finally, ref. [38] studies whether instantons melt into constituents in $CP^{(N-1)}$ models with the result that melting does not take place for $N \geq 3$.

Let me now mention some other instanton related works. In [39,40] first evidence for a stronger correlation between instantons and anti-instantons in dynamical configurations has been measured. This has relevance for the instanton liquid model which predicts I-A correlations in the presence of dynamical quarks [1]. In [41] the low-lying mesonic spectrum has been studied in ensembles of instantons with properties as obtained from the lattice, much in the spirit of the instanton liquid model. A first attempt to reproduce the real spectrum failed due to a mixture between physical states and free lattice modes. Removing these free modes by adding a perturbative background results in an excellent agreement. Finally, the controlled cooling technique developed in [42] to reduce uncertainties in the analysis of the instanton content of MC configurations has been extended to SU(3) [43].

4. Center vortices

The idea that center vortices might be relevant for confinement in Yang-Mills theories is also a very old one [16,17]. It is perhaps in 3 dimensions where it becomes most appealing [16]. The 3-D vortex is a topologically stable soliton of the theory. The vortex creation operator, ϕ, is a local field whose vev signals the spontaneous symmetry breaking of the Z_N magnetic symmetry, mapping $\phi \to e^{\frac{2\pi i n}{\mathrm{N}}}\phi$. This allows us to postulate an effective low-energy theory in terms of such a local field. In the broken phase there are N degenerate vacua, and the Wilson loop is the creation operator of the domain wall that separates them. This wall is stable because the vacuum surrounding it is. In this picture the string tension confining quarks is related to the tension of the wall.

An extension of these ideas to 4D is not straightforward. In 4D the vortex creation operator is no longer local (for a recent discussion see [44]). The 4-D soliton is string-like and the natural low-energy effective theory is a theory of strings. Still, one can, following 't Hooft [16], define quantised electric and magnetic Z_N fluxes. Consider Yang-Mills theory in a 4-D torus of periods l_μ. The gauge potential is periodic in x_μ up to a gauge transformation $\Omega_\mu(x)$. Univaluedness of A_μ implies

$$\Omega_\mu(x+l_\nu)\Omega_\nu(x) = e^{\frac{2\pi i n_{\mu\nu}}{\mathrm{N}}}\Omega_\nu(x+l_\mu)\Omega_\mu(x) \quad (3)$$

with twist $n_{\mu\nu}$ (defined mod N) allowed to be non-zero due to blindness of A_μ to Z_{N}. $m_i \equiv \frac{1}{2}\epsilon_{ijk}n_{jk}$ counts the magnetic flux in the box along direction i, while $k_i \equiv n_{0i}$ is dual to the electric flux e_i. Electric flux along a curve C is generated by the Wilson loop, $W(C)$. The 't Hooft loop operator, $B(C)$, non-local in the gauge fields, creates magnetic flux along C. 't Hooft argued that in the absence of massless particles, the vacuum should be in one of two phases parametrised by the vevs of W and B. In the Higgs phase they show respectively perimeter and area laws. The confined phase is dual to it and the roles of W and B are interchanged.

The free energy of a state of given $\vec{e}$ and $\vec{m}$ is

$$e^{-\beta F(\vec{e},\vec{m})} = \sum_{\vec{k}} e^{-\frac{2\pi i \vec{k}\cdot\vec{e}}{\mathrm{N}}} Z(\vec{k},\vec{m}) \quad (4)$$

with $Z(\vec{k},\vec{m})$ the partition function with twisted boundary conditions specified by $\vec{k}$ and $\vec{m}$. For simplicity I have assumed that the instanton θ angle is zero; the θ dependence gives rise to peculiar phenomena which will not be discussed here.

Euclidean symmetry gives rise to an exact electric-magnetic duality between free energies of different fluxes. In the presence of a mass gap such duality implies that, in the β, $l_i \to \infty$ limit, some of the fluxes have to be heavy. Moreover, either all the electric or all the magnetic fluxes are light, no mixing between them takes place. Suppose it is the electric fluxes which are heavy and confined in thin strings/vortices. Assuming that at large β the free energy factorizes in an electric, $F(\vec{e})$, plus a magnetic, $F(\vec{m})$, part one derives

$$F((m,0,0)) = 2\lambda\ (1-\cos(\frac{2\pi m}{\mathrm{N}}))\ l_1\ e^{-\sigma l_2 l_3} \quad (5)$$

with σ the string tension of the electric confining string. The free energy of magnetic flux decreases exponentially as we let the box become large in the plane transverse to the flux. Magnetic fluxes spread over the whole volume and condense. This behaviour parametrises the confinement phase. It is derived solely from duality and the existence of heavy electric fluxes. Indeed, duality does not tell whether it is the electric or the magnetic fluxes which condense. In the Higgs phase the roles of electric and magnetic fluxes are interchanged.

Implementing twisted boundary conditions on the lattice is rather easy. For SU(2) a twist $n_{\mu\nu} = 1$ can be enforced by flipping the sign of all the plaquettes sitting in, for instance, the upper right corner of each (μ,ν) plane. Notice that magnetic flux is only defined modulo 2, as is the number of twisted plaquettes per plane.

This year we have seen a revival of calculations of magnetic flux free energy on the lattice, both at zero [45,46] and at finite temperature [47–51]. Already in [52] it was proposed to use magnetic twist as a probe for phase structure. In order to compute the free energy of magnetic flux, one computes the ratio of two partition functions: $\exp\{-\beta F(\vec{m})\} = Z(\vec{m})/Z(\vec{0})$. Results at zero temperature [45,46] support the exponential behaviour indicated in (5). It would be nice to check if the coefficient of the exponential decay of the free energy does indeed agree with the electric string tension. At finite temperature there are results, both for 3 [51] and 4 dimensions [47–50], corroborating the dual behaviour of 't Hooft and Wilson loops. There is, for non-zero T, a difference between introducing the twist in space-time or space-space planes. As shown in [48] space-space 't Hooft loops show area law, corresponding to deconfinement of space-time Wilson loops. However, space-time 't Hooft loops are screened in agreement with the observation that the spatial string tension survives above T_c. Similar results are obtained in [47,51]. Previous analytic calculations of the expectation value of the 't Hooft loop at high temperature [53] also support this picture.

It is worth mentioning the comparison in [49,50] between disorder operators signalling monopole and magnetic flux condensation. Both of them behave in a very similar way giving a critical temperature compatible with the standard determinations.

Another issue much more difficult to settle, is whether indeed the disordering of Wilson loops is driven by the presence of thick magnetic vortices in the vacuum as advocated in [17].

Smooth vortex configurations do exist and have been obtained on the lattice from cooling [54]. For this the use of twisted boundary conditions is again essential. Here one remark is important. With twisted b.c. such that $\vec{k} \cdot \vec{m} \neq 0$ (mod N) the topological charge is fractional and quantised in units of 1/N. We have already discussed fractional charge objects in connection to monopoles. Vortices found in [54] also carry fractional charge.

Another nice connection between vortices and topology has been put forward in [55]. Based on a model that describes vortices as random surfaces [56], topology is incorporated by providing orientation to the surface. Non-trivial topological charge comes from non-globally-orientable surfaces describable as patches of equal orientation separated by monopole lines. A prediction for the zero temperature susceptibility of $\chi_q(T=0) = (190 \pm 15\text{MeV})^4$ is derived, in amazingly good agreement with lattice results - see sec. 2.

How to locate thick center vortices on lattice configurations has been the subject of a big debate this year. In [14] an approach very similar to 't Hooft's abelian projection was taken (other alternative approaches will not be discussed, for a recent review see [57]). Center vortices are located by fixing the so-called maximal center gauge (MCG), obtained by maximising the average of $|\text{Tr}(U_\mu(x))|^2$. Let us concentrate on the SU(2) case. Center projection consists of replacing gauge fixed links by the closest Z_2 element. Center-projected (P-)vortices correspond to closed sets of plaquettes taking value (-1). It is claimed that the string tension from center-projected links (σ_{Z_2}) agrees with the full string tension ($\sigma_{\text{SU}(2)}$), a phenomena dubbed as center dominance. The relevance of this is, however, obscured by the fact that center dominance appears to be obtained even without gauge fixing [58]. The physicality of P-vortices has to be judged on a different basis. Tests in [14] correspond to the behaviour of Wilson loops pierced by even/odd number of P-vortices, and to the scaling of the P-vortex density. The behaviour of P-vortices across the deconfinement phase transition has also been studied [59].

The news this year is that maximal center gauge turns out to be severely affected by lattice Gribov copies. A first indication in this direction was provided in [60]. If, prior to fixing MCG, the configurations are driven into a smooth gauge like the Landau gauge, center dominance is lost and the density of P-vortices dramatically reduced. Worrisome is that Landau preconditioning usually gives a higher local maximum than the one from direct MCG. Further evidence comes from [61,62] where several random copies of the same configuration are made, taking from them the one that gives the higher local maximum after MCG. The number of copies is extrapolated to infinity, with the result again that a very significant part of $\sigma_{\text{SU}(2)}$ is lost in center projection, even in the continuum limit. The debate originated about this issue (see [61,63]) seems settled in [62] with a very careful study of the dependence on the number of gauge copies and the results stated above.

One can do better by performing a gauge fixing free of lattice Gribov ambiguities. This is the case of the Laplacian gauge, first introduced for abelian projection [64] and further extended to perform center gauge fixing [65] (see also [66]). The idea is to diagonalise the adjoint Laplacian and use its two lowest eigenvectors to fix the gauge. Here instantons, monopoles and vortices arise respectively as point, 1- or 2-dimensional singularities of the gauge fixing [67]. Indeed, in the Laplacian gauge, center dominance is recovered, although only in the continuum limit. But we have by now repeatedly said that center dominance alone is not good enough. Further investigation on the vortex content of Laplacian gauge fixed configurations is still necessary.

5. Closing remarks

There is still a lot of work to do to unveil the mysteries of confinement. Perhaps the relative failure of the approaches taken so far is related to our insistence on describing confinement in 'semiclassical' terms. We tend to bear in mind that some underlying 'classical' fields (be it 'fat' monopoles, vortices or instantons) drive the phenomena. But attempts to identify them in nonperturbative ensembles have systematically led to problems. There might be some truth in it but the key ingredient seems to be still missing.

Acknowledgements

I thank the organisers for a very enjoyable conference. I am indebted to José Luis F. Barbón, Tony González-Arroyo and Pierre van Baal for invaluable discussions over the years. I would also like to thank Philippe de Forcrand, Adriano Di Giacomo, Jeff Greensite, Tamas Kovacs, Alvaro Montero, Carlos Pena and Nucu Stamatescu.

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